Math 62: Lesson #4 Appendix A
Division - especially
Synthetic Division

Math 72: Lesson #28 6.7 - review divide monomials
Division - review long division
and synthetic division

Math 70: 5.8 Division of Polynomials

Lesson 16

(Yes, we skipped 5.7)

5.8 Division of Polynomials

Objectives

1) Divide by a monomial
2) Divide by a polynomial
3) Use synthetic division to divide by a binomial
4) Know the limitations of synthetic division.

Long Division & Synthetic Division

Review Math 45 — 1) Divide polynomial by monomial

Review Math 45 — 2) Divide polynomial by polynomial using long division

* 3) Use synthetic division to divide a polynomial by a linear binomial with leading coefficient 1.

Divide

① $\dfrac{10x^3 - 5x^2 + 21x}{5x}$ ← polynomial / ← monomial (one term).

$= \dfrac{10x^3}{5x} - \dfrac{5x^2}{5x} + \dfrac{21x}{5x}$

write each term of the numerator as a separate fraction over the denominator

Signs between fractions come from the numerator.

$= \boxed{2x^2 - x + \dfrac{21}{5}}$

Reduce each fraction

② $\dfrac{3x^5y^2 - 15x^3y - x^2y - 6x}{2x^2y}$

$= \dfrac{3x^5y^2}{2x^2y} - \dfrac{15x^3y}{2x^2y} - \dfrac{x^2y}{2x^2y} - \dfrac{6x}{2x^2y}$

$= \boxed{\dfrac{3}{2}x^3y - \dfrac{15}{2}x - \dfrac{1}{2} - \dfrac{3}{xy}}$

CAUTION: The hardest part of these problems is remembering that this is the process!

Remember long division?

$$\begin{array}{r} 14 \\ 17\overline{)253} \\ -17 \\ \hline 83 \\ -68 \\ \hline 15 \end{array}$$

answer $14\frac{15}{17}$

$\frac{25}{17} = 1 +$ stuff

$1 \times 17 = 17$

subtract $25 - 17$

— repeat —

$\frac{83}{17} = 4 +$

$17 \times 4 = 68$

Polynomial long division is exactly the same.
(If you were taught to do the subtraction in your head, please don't do poly subtract in your head.)

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Divide using a) long division
b) synthetic division

③ $\dfrac{2x^2 - x - 10}{x+2}$

$$
\begin{array}{r|l}
 & 2x \\
\hline
x+2 & 2x^2 - x - 10 \\
 & \underline{2x^2 + 4x}
\end{array}
$$

$\dfrac{2x^2}{x} = 2x$

Always Line up like terms!

$2x(x+2) = 2x^2 - 4x$

To subtract polynomials, we usually write:

$(2x^2 - x) - (2x^2 + 4x)$

dist negative:

$= 2x^2 - x - 2x^2 - 4x$

and combine like terms:

$= -5x$

Now we're going to do this vertically by
- changing the signs
- adding like terms vertically

$$
\begin{array}{r|l}
 & 2x - 5 \\
\hline
x+2 & 2x^2 - x - 10 \\
 & \underline{-2x^2 - 4x} \\
 & \quad -5x - 10 \\
 & \quad \underline{+5x \;\; 10} \\
 & \qquad\quad 0
\end{array}
$$

Bring down -10 and repeat

answer: $\boxed{2x - 5}$

③ again, by synthetic division:

(box)			

empty structure: box, 2 blank lines, line below.

-2			

In the box, put the solution that corresponds to our divisor as a factor:

$x + 2 = 0$

$x = -2$

* changing the sign here takes care of all distribute-the-negative steps

-2	2	-1	-10

Across the first row, write the coefficients from the dividend, in standard form,

* using placeholder 0s for any missing terms

-2	2	-1	-10
	2		

Bring down the 1st coefficient unchanged, write it below the line

-2	2	-1	-10
		-4	
	2		

Multiply the number below the line by the number in the box, write result in next column, above the line.

-2	2	-1	-10
		-4	
	2	-5	

Add the numbers, put result below the line.

-2	2	-1	-10
		-4	10
	2	-5	0

Repeat until the end of the line.

③ Synthetic division, continued.

Notice that the degree of the dividend is reduced by one to get the quotient:

$$\begin{array}{r|rrr} -2 & 2x^2 & -1 & -10 \\ & & & \\ \hline & 2x & -5 & 0 \end{array}$$

← The remainder appears in the last place

NOTICE: The numbers above the line appeared above each subtraction line in our long division process.
We don't need the first terms because they always add to zero.

CAUTION: Synthetic division only works in very limited circumstances

- The divisor must be linear
- The leading coefficient of the divisor must be 1.

NC ④ Divide $\dfrac{6x^2 - 19x + 12}{3x - 5}$

must use long division because leading coef $\neq 1$

$$\begin{array}{r l} & \;2x \;\; -3 \\ 3x-5 \,\big) & 6x^2 - 19x + 12 \\ & -6x^2 + 10x \\ \hline & \qquad -9x + 12 \\ & \qquad +9x \mp 15 \\ \hline & \qquad\qquad -3 \end{array}$$

$$\frac{6x^2}{3x} = 2x$$

$$\frac{-9x}{3x} = -3$$

answer $\boxed{2x - 3 + \dfrac{-3}{3x-5}}$

or $\boxed{2x - 3 - \dfrac{3}{3x-5}}$

(4) Method 2: You can rewrite this so synthetic division can be used, but you must be comfortable with fractions!

$$\frac{6x^2 - 19x + 12}{3x - 5} \quad \frac{(\frac{1}{3})}{(\frac{1}{3})}$$

Want leading coef 1 in denom.

$$= \frac{\frac{6}{3}x^2 - \frac{19}{3}x + \frac{12}{3}}{\frac{3}{3}x - \frac{5}{3}}$$

$$= \frac{2x^2 - \frac{19}{3}x + 4}{x - \frac{5}{3}}$$

$$\begin{array}{c|ccc} \frac{5}{3} & 2 & -\frac{19}{3} & 4 \\ & & \frac{10}{3} & -5 \\ \hline & 2 & -3 & -1 \end{array}$$

GC [MATH] > Frac

which means $2x - 3 + \frac{-1}{x - 5/3}$ } cannot express remainder as a complex fraction

$$= 2x - 3 + \frac{-1\,(3)}{(x - \frac{5}{3})(3)}$$

$$= \boxed{2x - 3 - \frac{3}{3x - 5}}$$

same as before

(5) Divide $\dfrac{6x^4 - 3x + 10x^2}{x-5}$

Notice:
The numerator (dividend) is a mess.
- It's not in standard form.
- It's missing terms. $0x^3$ and $0x^0$ ← plain numbers, "constant term".

$$= \frac{6x^4 + 0x^3 + 10x^2 - 3x + 0}{x-5}$$

Denominator is a linear binomial with leading coefficient 1.

We can use either long division OR synthetic division

Long Division

$$\begin{array}{r} 6x^3 + 30x^2 + 160x + 797 \\ x-5 \overline{) \, 6x^4 + 0x^3 + 10x^2 - 3x + 0} \\ -6x^4 + 30x^3 \qquad\qquad\qquad \\ 30x^3 + 10x^2 \qquad\qquad \\ -30x^3 + 150x^2 \qquad\qquad \\ 160x^2 - 3x \qquad \\ -160x^2 + 800x \qquad \\ 797x + 0 \\ -797x + 3985 \\ 3985 \end{array}$$

$\dfrac{6x^4}{x} = 6x^3$

$\dfrac{30x^3}{x} = 30x^2$

$\dfrac{160x^2}{x} = 160x$

$\dfrac{797x}{x} = 797$

$$\boxed{6x^3 + 30x^2 + 160x + 797 + \frac{3985}{x-5}}$$

Synthetic Division

5	6	0	10	-3	0
		30	150	800	3985
	6	30	160	797	3985

$$\boxed{6x^3 + 30x^2 + 160x + 797 + \frac{3985}{x-5}}$$

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2nd

no (5) Divide $\dfrac{7x^3+16x^2+2x}{x+4}$

← linear, leading coef = 1

-4	7	16	2	0
		-28	48	-200
	7	-12	50	-200

← $x+4=0$
Solution -4 in box

answer $\boxed{7x^2-12x+50-\dfrac{200}{x+4}}$

Yes (6) Divide $\dfrac{3x^4+2x^3-8x+6}{x^2-1}$

BE CAREFUL!
- Need a placeholder for x^2
- Be sure to line up like terms

$$
\begin{array}{r}
3x^2+2x+3 \\
x^2-1\ \overline{)\,3x^4+2x^3+0x^2-8x+6} \\
-3x^4\qquad\quad +3x^2\qquad\quad \\
\hline
2x^3+3x^2-8x\quad \\
-2x^3\qquad\ +2x\quad \\
\hline
3x^2-6x+6 \\
-3x^2\qquad +3 \\
\hline
-6x+9
\end{array}
$$

Can't use synthetic because it's not linear (divisor).

$\dfrac{3x^4}{x^2}=3x^2$

$\dfrac{2x^3}{x^2}=2x$

$\dfrac{3x^2}{x^2}=3$

answer: $\boxed{3x^2+2x+3+\dfrac{-6x+9}{x^2-1}}$

(7) Divide $\dfrac{27x^3+8}{3x+2}$

← Can't use synthetic because leading coef isn't 1.

$$
\begin{array}{r}
9x^2-6x+4 \\
3x+2\ \overline{)\,27x^3+0x^2+0x+8} \\
-27x^3\mp 18x^2\qquad\qquad \\
\hline
-18x^2+0x\quad \\
+18x^2+12x\quad \\
\hline
12x+8 \\
-12x\mp 8 \\
\hline
0
\end{array}
$$

← Need placeholders for both x^2 and x.

$\dfrac{27x^3}{3x}=9x^2$

$\dfrac{-18x^2}{3x}=-6x$

$\dfrac{12x}{3x}=4$

answer $\boxed{9x^2-6x+4}$

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(7) con't.

You can rewrite this problem so that synthetic division is possible, but you will need to be comfortable using fractions.

$$\frac{27x^3+8}{3x+2}\cdot\frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)}$$ ← multiply by 1 (or divide all terms by 3)

$$= \frac{\frac{27}{3}x^3+\frac{8}{3}}{\frac{3x}{3}+\frac{2}{3}}$$

$$= \frac{9x^3+\frac{8}{3}}{x+\frac{2}{3}}$$ ← now denom is linear with leading coefficient 1.

$$\begin{array}{r|rrrr} -\frac{2}{3} & 9 & 0 & 0 & \frac{8}{3} \\ & & -6 & 4 & -\frac{8}{3} \\ \hline & 9 & -6 & 4 & 0 \end{array}$$

← MATH > FRAC is your friend

$$\boxed{9x^2-6x+4}$$ same as we got before

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Note ⑦ was a way to remember the trinomial factor for the sum of cubes...

⑧ Divide $\dfrac{2x^3 - x^2 - 13x + 1}{x - 3}$

3	2	−1	−13	1
		6	15	6
	2	5	2	7

answer $\boxed{2x^2 + 5x + 2 + \dfrac{7}{x-3}}$

⑨ Divide $\dfrac{x^4 - 2x^3 - 11x^2 + 5x + 34}{x+2}$

−2	1	−2	−11	5	34
		−2	8	6	−22
	1	−4	−3	11	12

answer $\boxed{x^3 - 4x^2 - 3x + 11 + \dfrac{12}{x+2}}$

⑩ $\dfrac{3x^2 - 4}{x - 1}$

1	3	0	−4
		3	3
	3	3	−1

← Don't forget placeholder!

answer $\boxed{3x + 3 + \dfrac{-1}{x-1}}$

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(11) $\dfrac{x^4 - \frac{2}{3}x^3 + x}{x - 3}$

3	1	-2/3	0	1	0
		3	7	21	66
	1	7/3	7	22	66

← Need placeholders for both x^2 and constant terms

← Use GC with >frac!

answer $\boxed{x^3 + \frac{7}{3}x^2 + 7x + 22 + \frac{66}{x-3}}$

(12) $\dfrac{2x^5 + x^3 + 3 - 6x^4 - 4x}{x^2 - 3}$

← out of order! and x^2 needs placeholder!

← can't use synthetic because it's not linear

$$
\begin{array}{r}
2x^3 - 6x^2 + 7x - 18 \\
x^2 - 3 \,\big)\, 2x^5 - 6x^4 + x^3 + 0x^2 - 4x + 3 \\
-2x^5 \qquad\quad + 6x^3 \qquad\qquad\qquad \\
\hline
-6x^4 + 7x^3 + 0x^2 \qquad\qquad \\
+6x^4 \qquad \mp 18x^2 \qquad\qquad \\
\hline
7x^3 - 18x^2 - 4x \qquad \\
-7x^3 \qquad\quad + 21x \qquad \\
\hline
-18x^2 + 17x + 3 \\
+18x^2 \qquad\quad \mp 54 \\
\hline
17x - 51
\end{array}
$$

$\frac{2x^5}{x^2} = 2x^3$

$\frac{-6x^4}{x^2} = -6x^2$

$\frac{7x^3}{x^2} = 7x$

$\frac{-18x^2}{x^2} = -18$

answer: $\boxed{2x^3 - 6x^2 + 7x - 18 + \frac{17x - 51}{x^2 - 3}}$

"Simplify your answer" means "reduce all fractions."

"Do not factor" means "Do not do this problem by factoring and canceling."

6.4.87 Divide.

$(14x^4 - 4x^2 + 21x^3 - 6x) \div (14x + 21)$

$(14x^4 - 4x^2 + 21x^3 - 6x) \div (14x + 21) = x^3 + -\frac{2}{7}x$

(Simplify your answer. Do not factor.)

Should be done by long division, or with adjustment, by synthetic division.

Long Division:

$$14x + 21 \overline{)\, 14x^4 + 21x^3 - 4x^2 - 6x + 0} \quad \text{quotient: } x^3 \;\; -\frac{2}{7}x$$

$$\mp 14x^4 \mp 21x^3$$

$$0 - 4x^2 - 6x$$

$$\pm 4x^2 \pm 6x$$

$$0 + 0.$$

$14x(?) = -4x^2$

$(?) = \frac{-4x^2}{14x} = \frac{-2}{7}x$

solution: $\boxed{x^3 - \frac{2}{7}x}$

Synthetic Division:

$$\frac{\frac{1}{14}(14x^4 + 21x^3 - 4x^2 - 6x + 0)}{\frac{1}{14}(14x + 21)}$$ ← need coefficient of x to be 1. Use GC ▷frac to do multiplying.

$$= \frac{x^4 + \frac{3}{2}x^3 - \frac{2}{7}x^2 - \frac{3}{7}x + 0}{x + \frac{3}{2}}$$

← $x + \frac{3}{2} = x - (-\frac{3}{2})$

Put $-\frac{3}{2}$ in box.

x^4 ↓

$-\frac{3}{2}$	1	$\frac{3}{2}$	$-\frac{2}{7}$	$-\frac{3}{7}$	0
		$-\frac{3}{2}$	0	$+\frac{3}{7}$	0
	1	0	$-\frac{2}{7}$	0	0

↑ x^3 one degree less.

solution $\boxed{x^3 - \frac{2}{7}x}$

A different problem. (Notice the instructions)

Simplify

$$\frac{14x^4 + 21x^3 - 4x^2 - 6x}{14x + 21}$$

$$= \frac{7x^3(2x+3) - 2x(2x+3)}{7(2x+3)}$$

$$= \frac{(7x^3 - 2x)\cancel{(2x+3)}}{7\cancel{(2x+3)}}$$

$$= \boxed{\frac{x(7x^2 - 2)}{7}}$$

Math 70 M-G 4/e 6.4 Long Division & Synthetic Division

Review M45 1) Divide polynomial by monomial

Review M45 2) Divide polynomial by polynomial

* 3) Use synthetic division to divide a polynomial by a binomial.

[(4) We are not doing the Remainder Theorem]

① Divide $\dfrac{10x^3-5x^2+21x}{5x}$

② Divide $\dfrac{3x^5y^2-15x^3y-x^2y-6x}{2x^2y}$

Extras

*③ Divide $\dfrac{2x^2-x-10}{x+2}$ a) long division b) synthetic division

④ Divide $\dfrac{6x^2-19x+12}{3x-5}$ a) long division b) why not by synthetic?

*⑤ Divide $\dfrac{7x^3+16x^2+2x-1}{x+4}$ a) by synthetic b) why bother with long?

⑥ Divide $\dfrac{3x^4+2x^3-8x+6}{x^2-1}$

⑦ Divide $\dfrac{27x^3+8}{3x+2}$

* ⑧ Divide $\dfrac{2x^3-x^2-13x+1}{x-3}$ (by synthetic)

*⑨ Divide $\dfrac{x^4-2x^3-11x^2+5x+34}{x+2}$

Extras:

⑩ $\dfrac{3x^2-4}{x-1}$

⑪ $\dfrac{x^4-\frac{2}{3}x^3+x}{x-3}$

} synthetic w/ missing term.

⑫ $\dfrac{2x^5-6x^4+x^3-4x+3}{x^2-3}$ } mixed up terms long division